Designing a Mobile Mathematics Application for Prospective Elementary School Teachers

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To know mathematics is to possess not a singular entity, but a multidimensional construct. What one knows of mathematics is dependent upon what one has been taught, what the experience of that teaching was, and the subsequent use of the mathematics learned. It is generally recognized that students in the United States have a weak knowledge of mathematics. The manifold impact of weakness in mathematics knowledge is particularly salient when the students under consideration are undergraduate prospective elementary school teachers.

Teacher Knowledge

It is unsurprising that the mathematical knowledge that an undergraduate prospective elementary teacher must acquire to adequately teach is more specialized than the mathematical knowledge of the general public. Mathematical knowledge for teaching falls into two categories: content knowledge (CK) and pedagogical content knowledge (PCK). These aspects of teacher knowledge were defined by Shulman (1986) in his landmark article that separated the two types of knowledge in a way that educational researchers still use to investigate teacher content knowledge. Shulman defined content knowledge as "going beyond knowledge of the facts or concepts of a domain. It requires understanding the structures of the subject matter" (pg. 9). Content knowledge includes knowledge of the facts and procedures of mathematics and the understanding of the purpose and relationships within mathematics and beyond mathematics, containing both theory and practice. PCK is the specific forms of content knowledge related to how mathematics is taught. PCK includes alternate ways in which CK can be represented to make it understandable to students, knowledge of the preconceptions and misconceptions that students bring to the subject, and strategies for organizing student understanding. Shulman identified curricular knowledge as a third type of teacher knowledge that is beyond the scope of this paper.

Research indicates that there is a historical and ongoing weakness in preservice teacher mathematics knowledge. A longitudinal study of CK (Ball, 1990) of 252 preservice teacher candidates at the point in which they entered formal teacher education found that, instead of the connected mathematics knowledge described by Shulman (1989), many emerged from their content coursework with skills that were limited to discrete procedural processes disassociated from larger mathematical concepts. In addition, many of the prospective teachers focused on procedures and rules because they perceived the process of doing mathematics as simply following set procedures step-by-step to generate answers: that mathematics itself was an arbitrary collection of facts and rules to be remembered and employed. The author argued that the data suggested “that the mathematical understandings that prospective teachers bring are inadequate for teaching mathematics for understanding,” (Ball, 1990, p. 464). A decade later, in spite of curriculum reforms and robust professional development programs, weakness in mathematics content knowledge had not improved. A review of research on teacher knowledge and teachers’ use of mathematical knowledge (Ball, Lubienski, & Mewborn, 2001) of studies that looked closely at teacher’ knowledge of multiplication, division, rational numbers, functions, geometry, and proofs found pervasive weaknesses in U.S. teachers’ understanding of mathematical ideas and relationships.

In the early 2000s an effort was begun to develop measures that could empirically test mathematical content knowledge that teachers possess (Hill, Ball, & Schilling, 2004). The researchers sought to develop a construct that represented mathematical knowledge for teaching (MKT). They pilot tested numerous multiple-choice items intended to represent the mathematical knowledge used in teaching elementary mathematics with 1,552 elementary school teachers. Exploratory factor analysis found three underlying dimensions of MKT: knowledge of content in number concepts and operations, knowledge of content in patterns, functions, and algebra, and knowledge of students and content in number concepts and operations. In addition, they found that specialized knowledge included knowledge of specialized tasks specific to teaching and knowledge of student. They concluded that MKT consists of more than the knowledge of mathematics held by a well-educated adult; there was evidence of more mathematical depth to teaching elementary school. They found that MKT was a multiple-dimensional construct that consisted of a strong knowledge of basic mathematics that provided a foundation for the specialized knowledge that teachers use to teach. The authors state that the additional knowledge may include teacher understanding of why mathematical statements are true and knowledge of multiple representations of mathematical ideas.

As part of the ongoing development of measures to test MKT, Hill, Ball, and Schilling (2008) developed a framework that refined Shulman's (1989) division of teacher knowledge into subdomains (Figure 1). It is important to note that the measure of MKT incorporates both subject matter with no knowledge of students or teaching entailed, and Shulman's proposed PCK. The three portions of the oval under Subject Matter Knowledge include mathematics knowledge that does not include knowledge of students or teaching. Common content knowledge (CCK) is mathematics to be taught that used in professions or occupations. Specialized content knowledge (SCK) is the knowledge of how to represent mathematical ideas, and explanations or common rules and procedures. Horizon knowledge is an awareness of how mathematical topics are related over the span of mathematics (Zazkis & Mamolo, 2011). The three portions under the right side of the oval under Pedagogical Content Knowledge include knowledge of content and students (KCS), knowledge of content and teaching (KCT), and knowledge of curriculum.

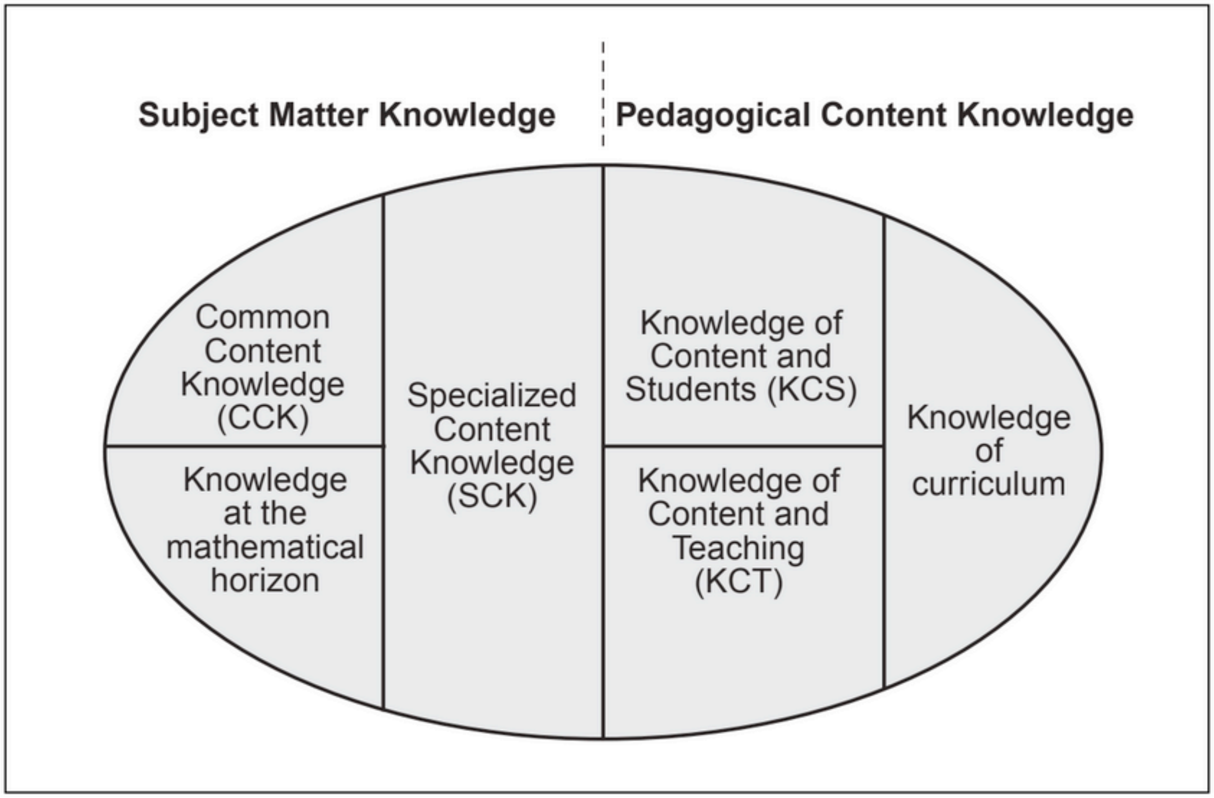

Figure 1. Domain map for mathematical knowledge for teaching (Hill, Ball, & Schilling, 2008).

A study (Hill, Ball, and Schilling, 2008) focused specifically on the development of a measure of knowledge of content and students (KCS), a subset of PCK which is a subset of MKT, was conducted using a pretest/posttest design of 640 teachers attending a number and operations professional development course for elementary teachers. This was followed by interviews of 26 K-6 teachers from three Midwestern school districts who were selected on the basis of either low or high CK scores. The researchers found that teachers relied both on familiarly with student error and on mathematical analysis to answer questions correctly. The multidimensionality demonstrated led to problems in measurement of KCS because researchers found that mathematical reasoning and knowledge could compensate for a lack of KCS leading to false positives on the measurement of KCS. Differentiation of the multidimensional aspects of the mathematical knowledge for teaching remained elusive.

Though differentiated understanding of teacher knowledge was obscured by high levels of mathematical reasoning and knowledge in some participants, the participants' ability to compensate for an absence of knowledge students potentially may shed light on what foundational knowledge effective mathematics teachers possess. In the years since, Shulman (1989) defined the concept of mathematical knowledge that is specific to teaching, a large body of work investigating mathematics teacher knowledge has accumulated. The Hill, Ball, and Schilling (2008) subdivisions of Shulman's SMK and PCK have become the most commonly used measures of teachers' mathematical knowledge (Blömeke & Delaney, 2012). A meta-analysis of 60 research articles that investigated PCK (Depaepe, Verschaffel, & Kelchtermans, 2013). All of the studies connected content knowledge and pedagogy. The authors found that conceptualizations of PCK fell into two distinct categories. The first category approached PCK from a cognitive perspective that had provided empirical evidence for a positive connection between teachers' PCK and student learning outcomes. The second category of studies approached PCK from a situated perspective that provided insight into what actually happens in classrooms. In all studies PCK was seen as a form of practical knowledge and content knowledge was described as an important and necessary prerequisite.

Effects on Student Achievement

Measuring teacher content knowledge would be meaningless without an understanding of what how MKT may impact student learning. A study of 115 elementary schools **(**Hill, Rowan, & Ball, 2005) investigated how teachers' MKT contributes to students' mathematics achievement. Data included student assessments administered in the fall and spring of each academic year. Teacher data was gathered with a highly structured self-reported log of the time devoted to mathematics instruction, content covered, a survey questionnaire of educational background, involvement in professional development, and language arts and mathematics teaching. The survey was the source of items included in the content knowledge for teaching mathematics measure. The measure was composed of multiple-choice items representing teaching-specific mathematical skills. The study found that teachers' mathematical knowledge for teaching positively predicted student gains in mathematics achievement. The authors conclude that content-focused professional development and preservice programs will improve student achievement and recommend a study of how knowing mathematics affects instruction.

Another study investigated the relationship between secondary teachers' MKT and student learning outcomes (Hatisaru & Erbas, 2017) using the framework developed by Hill, Ball, and Schilling (2008) to analyze two mathematics teachers and their ninth-grade students (*n=59*) in a vocational high school. The teachers were selected as representing strong and weak knowledge of functions. Measure of MKT was adapted from items used to measure student's knowledge of functions and teachers' knowledge of functions. Follow-up interviews were conducted to obtain a more detailed picture of knowledge of the function concept. The same instrument was used to test student outcomes at the end of a 5 week functions instructional unit. Classroom observations and follow-up interviews were conducted to discuss overall reaction to the class and planning for the next class. The researchers selected key aspects of the MKT that assessed specialized content knowledge (SCK), knowledge of content and students (KCS), and knowledge of content and teaching (KCT). The authors developed a concept framework for evaluating the knowledge domain and used their framework to conduct an qualitative analysis of the data gathered throughout the study. The results indicated that SCK was a necessary condition for KCS, and both aspects influenced instructional practices and learning outcomes.

A recent study (Tchoshanov et al., 2017) of 90 late elementary teachers (grades 5-9) and their students (*N=6,478),* looked closely at the components of MKT and the relationship with student achievement. Rather than using the construct developed by Hill, Ball, and Schilling (2008), the connection between MKT and the impact on student performance was studied using an instrument that assessed different cognitive types of teacher knowledge. Cognitive types were divided into knowledge of facts and procedures, including memorization and basic mathematical facts, rules, and algorithms (T1), knowledge of concepts and connections (T2), and knowledge of models and generalizations, conjecturing, generalizing, proving theorems (T3). The authors considered these types as low, medium, and high level knowledge types respectively. Student performance was measured using an end-of-course exam. A correlation found between teachers' content knowledge and student performance with teacher's overall mastery of content knowledge significantly associated with students attaining higher grades in mathematics classes. T1 and T2 were significantly correlated. T3 was not. Strength in T1 appeared to be necessary for high performance at the T2 level.

Foundational Knowledge

In spite of the difficulties of measuring teacher knowledge, across the studies, the consistent findings were that knowledge for teaching mathematics is dependent upon a foundational knowledge of integrated mathematics. Lipng Ma's (1999) book explores the disparity between Chinese and American student outcomes, with Chinese students typically outscoring U.S. Students on international comparisons of mathematics competency. Surprisingly, U.S. teachers have as much as 6 mores years of formal education than their Chinese counterparts.

Ma analyzed the fundamental mathematics that Chinese and American elementary teachers bring to their teaching and found a striking contrast between the nature of the knowledge between teachers in the two countries. U.S. teachers tended to be procedurally focused, competent with whole numbers, but challenged by more advanced topics such as division of fractions and perimeter and area of rectangles. In comparison, she found that Chinese teacher knowledge was "coherent while that of the U.S. teachers was clearly fragmented." (p. 107). Chinese teachers described why algorithms worked and when discussing alternative ways to solve problems explained that alternate approaches were possible, not because of isolated rules, but because of the underlying relationships that connect operations. Conceptual and procedural topics were interwoven and provided the groundwork to build further mathematics understanding. Ma identified this type of mathematical content knowledge as a *profound understanding of fundamental mathematics* (PUFM). Teachers who possess PUFM are able to conceive of the ideas that are connected to the structure of the discipline of mathematics. When challenged to demonstrate why an algorithm works they used not only verbal explanations and examples, they also justified their explanations with symbolic derivations.

Procedural and Conceptual Knowledge Frameworks

In order to study mathematics understanding and learning, a conceptual framework with definitions and meanings that are clearly understood within the mathematics research community must be established. A well-defined framework allows for meaningful dialogue and understanding across the discipline. Research can be designed around the framework, outcomes can be measured according to the operationalization of the components of the framework, and results can be understood within a shared conceptualization of the components. Within mathematics education research literature a well established and often used framework parses mathematical knowledge into two domains: procedural knowledge and conceptual knowledge. The conceptual and procedural knowledge framework was defined by Hiebert and Lefevre (1986) in their landmark article to provide a useful way for researchers to understand student learning processes. They acknowledged that the relationship between procedural knowledge and conceptual knowledge was not well understood, and that often mathematics knowledge may be an inseparable combination of both forms of knowledge. In spite of this, they argued that distinguishing types of knowledge would provide a way to understand the failure or success of building mathematics understanding.

Hiebert and Lefevre (1986) define conceptual knowledge as a connected web of knowledge: a network of linked relationships that are as important as the discrete elements. The web of knowledge is conceptual only if the learner recognizes the relationships between elements. Conceptual knowledge not only describes the connections between known elements, it also describes the connections made between existing knowledge and newly acquired knowledge. The context is created by the network of relationships, allowing the learner the freedom to apply existing knowledge to novel problems.

Procedural knowledge as described by Hiebert and Lefevre (1986) is the use of the formal language and the symbolic representation system of mathematics. Algorithms or rules are used to complete tasks with an awareness of only surface features. The knowledge of the meaning of the processes is not accessed or necessary for successful completion of tasks. Step-by-step instructions prescribe how to complete tasks. These tasks may then be sequenced into superprocedures that incorporate lower level subprocedures. Procedural knowledge allows students to solve complex superprocedures as a chain of prescriptions without knowledge of the meaning of the task.

In a response to Hiebert and Lefevre's (1986) bifurcation of mathematical knowledge into conceptual and procedural types, Star (2005) redefined conceptual knowledge as knowledge of concepts, principles, and definitions; and procedural knowledge as knowledge of procedures, algorithms, and the sequence of steps used in problem solving. He argued that contrary to the common perception of procedural knowledge as superficial and rote and the perception that conceptual knowledge is knowledge that is known deeply, that both types of knowledge can be either superficial or deep or anything in between. He described the depth of knowledge as *knowledge quality.* Using this alteration of the conceptual/procedural framework, two studies focused on flexibility in procedural knowledge. Researchers in both studies interpret flexibility in procedural processes, or knowledge of multiple strategies, as an indication of deep procedural knowledge. Looking at procedural knowledge without reference to conceptual acknowledges the importance of procedural knowledge as valuable in and of itself (Star, 2005).

The first study (Maciejewski & Star, 2016) was a teaching intervention designed to promote flexibility in procedural knowledge in first year undergraduate calculus students. The researchers sought to determine not only if procedural flexibility could be developed, but also if it resembled expert-like procedural performance. The design was quasi-experimental pretest/post test. Two sections of an introductory calculus course taught by the same instructor were selected for the experimental study. A pretest on differentiation was given. After receiving a lesson that contained a traditional sequence of instruction on procedures, the control section was given a typical worksheet for homework. The treatment section was given a worksheet that specified two approaches for solving each assigned problem and were asked to describe which method they prefered. The homework assignment was followed by a readministration of the pretest as a post test.

An analysis of the data found that here were no significant differences in the sections' score averages; both groups demonstrated higher achievement on the posttest. The treatment group used a greater variety of strategies than the control group and moved closer to expert-like performance. The authors concluded that it is possible to use an instructional task to support the development of undergraduate students' flexible use of procedures. Because the control group did not demonstrate flexibility after practice, the authors determined that an activity that prompted critical reflection by presenting tasks that prompt students to resolve questions in different ways and allow for the comparison of different solutions may support the development of deep procedural knowledge.

A second study (Lamb, Bishop, Philipp, Whitacre, & Schappelle, 2016) used clinical interviews to investigate the relationship between student flexibility in procedural problem solving and mathematics performance in students grades 2, 4, 7, and 11. The researchers sought to determine the degree in which flexible ways of reasoning influenced performance on integer problems. The wide grade span was chosen to cover a wide range of student learning experiences: from those who had not yet received school-based integer instruction to students who were enrolled in precalculus or calculus courses and therefore deemed to be successful high school mathematics students. Individual clinical interviews were conducted and videotaped at the students' school sites. The interviews were standardized and all students were asked to complete the same 25 open number sentences. Interviews were coded both for underlying reasoning and for correctness. Five broad categories and 41 subcodes provided detail into student's specific strategies. The five categories broken down into ways of reasoning: order-based, analogy-based, computational, formal, and developmental. Flexibility was measured by the variety of methods students used to solve integer-arithmetic tasks. Proficiency with a particular form of reasoning was demonstrated when a student used it three or more times. The number of ways that students used forms of reasoning that they were proficient in was the measure of flexibility.

Case studies were performed on three 7th grade students who exemplified the relationship between flexibility and accuracy. The first student chosen completed 32% of the open number sentences correctly. The second student completed 64% of the problems correctly. The third student completed 100% of the problems correctly. The researchers had found that the seventh grade students had had the greatest spread in flexibility. Eighty-five percent of 11th graders had used 3 or 4 methods. Flexibility scores for both groups had correlated positively with performance. This result also held across the case studies. The first and second cases used one type of reasoning almost exclusively which limited their options for solving problems and negatively impacted their success. The third case, who had completed every open number sentence correctly, flexibly used a wide range of strategies on the problems and appeared to choose strategies that corresponded with the underlying structure of the open number sentence.

Case studies provided insight into the relationship between flexibility and performance on open number sentence problems. The authors concluded that students who rely on a single way of reasoning may be impeded in their success because of their limited flexibility and that multiple ways of reasoning promotes successful performance. For every participant group, the correlation between flexibility and accuracy held; more flexible students were more successful.

Though both studies (Lamb et. al., 2016; Maciejewski & Star, 2016) addressed flexibility in procedural knowledge, neither study used the language of the conceptual and procedural knowledge framework, both studies explored the impact of deep procedural knowledge (Star,

2005) exemplified by procedural flexibility. Depth of conceptual knowledge was implied in the Maciejewski and Star (2016) study within the discussion of students' process of which method to use to solve problems for those students who used multiple methods. In the Lamb et. al. (2016) study both the clinical interviews and the more in-depth case study interviews provided insight into the procedural and conceptual understanding of the participants. Participant responses allowed the researchers to determine not only the presence of reasoning types but also the manner in which they were chosen. The interview process uncovered participants' procedural and conceptual knowledge, whether solutions were pursued through rote processes or driven by connected knowledge (Hiebert and Lefevre, 1986) and the depth, or lack of depth, of the participants' procedural and conceptual knowledge (Star, 2005).

The Exploration of Knowledge Embedded in Mathematical Symbols

High performance in teachers' Tchoshanov et al.'s (2017) cognitive types, T1 (knowledge of facts and procedures) and T2 (knowledge of concepts and connections), were associated with higher student achievement and align with the mathematical constructs explored in the studies of procedural flexibility (Lamb et. al., 2016; Maciejewski & Star, 2016) and other studies that have examined the iterative relationship between conceptual and procedural knowledge (Canobi, 2009; McNeil et al., 2012; Rittle-Johnson & Alibali, 1999). Across these studies conceptual knowledge was seen as the understanding of the underlying mathematical constructs represented by equations and mathematical expressions. Participants' conceptual knowledge was measured by the analysis of their responses to the symbolic representation of mathematical constructs. Whether assessing procedural or conceptual knowledge, the focus was on symbolic representations. The position of these studies was that student work traditionally described as procedural (Hiebert & Lefevre, 1986) potentially held conceptual understanding that manifested through transfer of knowledge to novel problems (Canobi, 2009; Rittle-Johnson & Alibali, 1999), or the ability to describe the underlying meaning of mathematical symbols (Canobi, 2009; Lamb, et. al., 2016; McNeil et al., 2012).

The challenges that prospective elementary teachers face in developing a sufficient knowledge base to be an effective teacher of mathematics has been studied with a great deal of talent and energy. In the effort to parse the complexities of learning and teaching mathematics, researchers have consistently returned to the foundational base of content knowledge that is presumed to be present in well-educated professionals. It is clear that adequate content knowledge must consist of more than the ability to solve problems. The knowledge must be deep and referential. The mathematics knowledge that emerges from these studies is practical, connected, and multifaceted.

Mobile Learning Interventions

If the primary strength of undergraduate mathematics knowledge is in procedural knowledge, then it makes sense to use students' existing procedural knowledge as a tool to build conceptual understanding. Rather than perceiving shallow procedural knowledge of undergraduate students as a liability, perhaps it can function as the starting point, the building blocks that can be used to create deep procedural knowledge by providing the opportunity for students to build the net of knowledge that interconnects the disparate parts of their existing mathematics understanding. The affordances of mobile learning environments have the potential to help students build those connections. Sequenced instruction that can build both procedural fluency and deep procedural knowledge as well as consistent practice may help students build the mathematics foundation that will inform the knowledge for teaching that they will continue to build in their teaching practice courses and classroom experiences.

Mobile learning tools can create opportunities for learning and practice in a student-controlled environment that allows users to navigate between topics and disciplinary content outside the formal classroom environment (Wang, Wiesemes, & Gibbons 2012). Mobile applications are popular as complementary tools for skills practice that are convenient to use because they do not demand long attention periods and can be used anywhere (Kinash, Brand, & Mathew, 2012; Sendurur et al., 2017). Immediate feedback on performance can create a game-like environment that can be motivating. It is important to note that a reward/punishment environment may discourage some learners (Sendurur et al., 2017). In this learner-driven environment scaffolding is necessary to support self-direction (andragogy) and self-determination (heutagogy) potentially leading to mobile learning adoption (Mac Callum, Jeffery, & Kinshuk, 2015).

Purpose of this Study

Hill, Ball, and Schilling (2008) found that mathematical reasoning and knowledge can compensate for an absence of teachers' knowledge of content and students. A meta-analysis of 60 research articles that investigated pedagogical content knowledge (Depaepe, Verschaffel, & Kelchtermans, 2013) determined that while PCK deals with a teacher's knowledge of the connection between content and pedagogy in a particular subject matter, content knowledge was an important and necessary prerequisite. What emerges from the research is that while both content knowledge and the knowledge of teaching are necessary for the preparation of prospective teachers, assuring that a foundational base of content knowledge must exist *first.* It is evident that the procedural and conceptual knowledge that will give prospective teachers the depth of knowledge that is necessary to teach others is not being developed in the current courses taught in secondary and undergraduate mathematics. This project will aide in the development of a self-study instrument that will allow students to make the connections, track their progress, and understand the landscape of conceptual mathematics foundational to the mathematical concepts taught in elementary schools.

Because of the importance of content knowledge to PCK, the purpose of this study will be to develop a mobile application which incorporates near-transfer sequencing of problems to develop procedural fluency in undergraduate elementary education majors. The study will add to the existing body of knowledge about preservice undergraduate prospective elementary school teachers' mathematics knowledge, and will develop an instrument to enlarge their connected knowledge base.

The following research questions will be investigated:

1. Can a mobile technology intervention improve mathematical procedural fluency in undergraduate prospective elementary teachers?
2. What motivates mobile learning adoption and ongoing engagement?
3. What elements of the mobile learning environment create stress in the user that results in disengagement?
4. Does use of a mobile application increase measures of self-determination?

Design Research Methodology

The design research model allows for a systematic approach to the development of an intervention and its impact in the challenging uncontrollable environment of mobile learning (Bannan, B., Cook, J., & Pachler, N., 2015). The presence of a learning application on a users mobile device has a potential impact on the users' perception of the device, user experience, the content, and the environment. Within this complex setting, design research provides rigorous cycles of applied research to "effect change in a learning context through the building of a design intervention through which we uncover pedagogical principles that may be applicable and researchable in similar situations" (Bannan, B., Cook, J., & Pachler, N., 2015, p. 3). As a qualitative method, design research allows for the uncovering of analytic generalizations from the specifics of the phenomenon being investigated.

Figure 2 (as cited in McKenney & Reeves, 2012, p. 16) illustrates the recursive nature of the design-based research process. Each loop represents a cycle of research with each cycle informing the direction and design of each succeeding cycle.

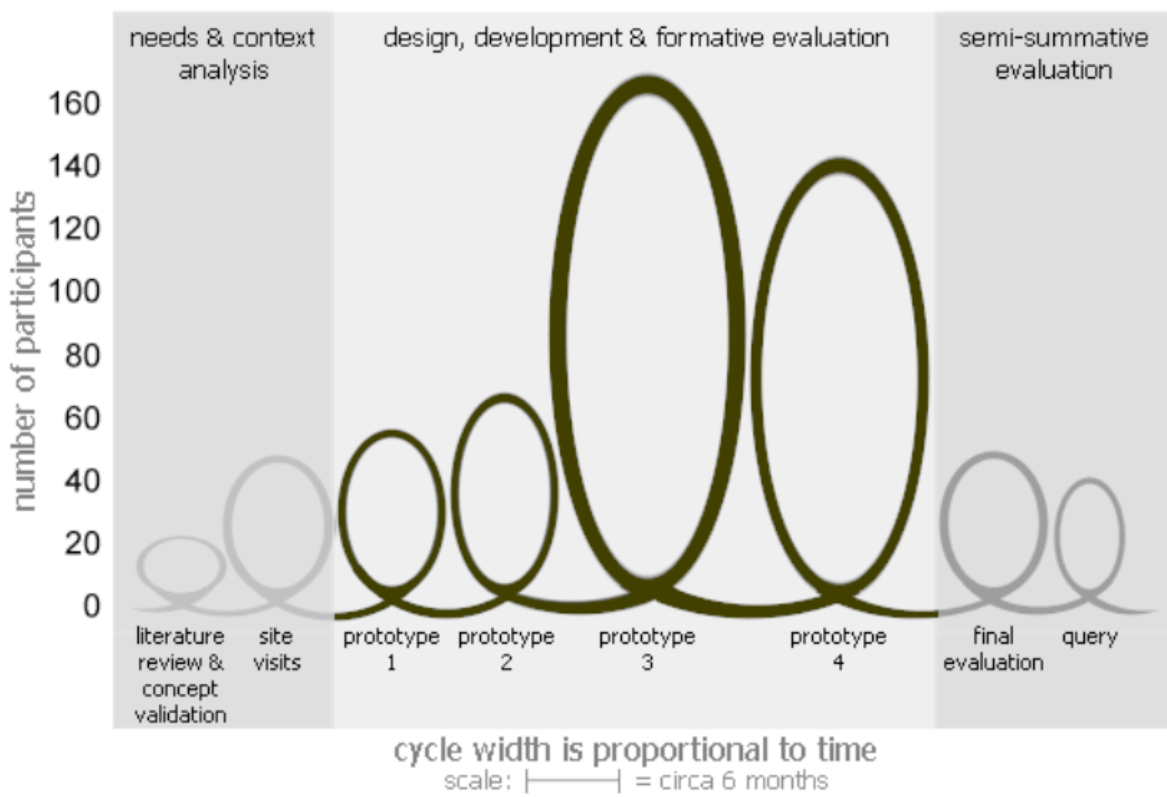

Figure 2. (McKenney & Reeves, 2012, p. 16)

The McKenney and Reeves (2012) framework describes the workflow of a specific research study and provides a clear organizing framework for the progression of development of a design based research study. While the looping form illustrates recursive design/research cycles, the direction of progress is continuously forward. Bannan's ILD Framework (Figure 3) shares the conceptual organization of the McKenney & Reeves with the Informed Exploration phase covering the needs and content analysis; the Enactment phase incorporating design, development, and formative evaluation; and the Enactment: Local Impact phase aligning with semi-summative evaluation. Bannan's Evaluation: Broader Impact phase moves beyond the project development process covered by the McKenney and Reeves framework and propels the project into real-world. In contrast to the McKenney and Reeves framework, the ILD Framework provides multiple avenues of exploration and research that will allow the trajectory of the project to change as more knowledge and experience is gained through the recursive research process.

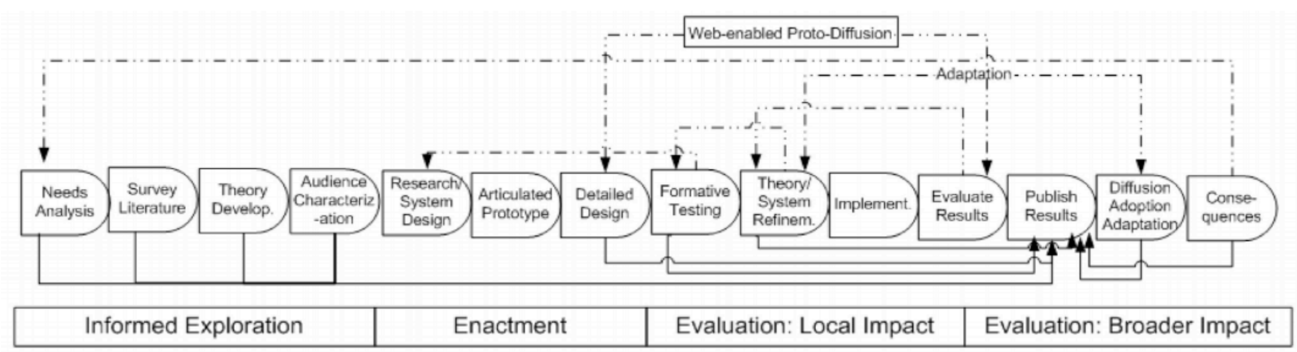

Figure 3. ILD Framework (Bannan-Ritland, 2003, p. 22)

The project will use the McKenney and Reeves model to sequence workflow for the project. The ILD Framework application mobile learning development, Bannan, Cook, and Pachler (2015) that presents guiding questions and applicable research methods for each of the phases will be used to design and research cycles and determine the trajectory of the research. The following is a preliminary plan for guiding questions and research methods for each phase of development of this project. While the McKenney and Reeves framework will guide the sequence of the research process, the ILDF will define the theoretical concepts being investigated and the choice of research methodologies that will be used within each cycle of research. Because of the qualitative nature of this research and the way in which prior cycles will influence the trajectory of subsequent cycles, the research design is expected to change over the course of project development.

Informed Exploration

At the beginning of the project the challenge will be to understand the target population and the learning experiences that have informed their knowledge base. The guiding questions for the Informed Exploration phase will be:

- What do we currently understand about the nature of the mathematics knowledge of undergraduate prospective elementary school teachers?
- How does this population build their mathematics knowledge?
- How do we measure mathematics knowledge for teaching in this population?
- How effective have mobile learning technologies been in building mathematics knowledge?

The foundation that the project will be built upon will start with what is currently known and what can be learned from the population and practitioners. The research methodologies used to investigate the guiding questions will be:

- A review of the existing literature.
- An ethnography of an undergraduate mathematics for elementary teachers course.
- Interviews with SMEs in undergraduate mathematics teaching and learning.

Enactment

The enactment phase embodies McKenney and Reeves (2012) microcycles of development and will consist of multiple cycles of design and research that will be used to refine the application. The initial prototype will be developed using the information gained from the Informed Exploration phase. Changes to the prototype will be guided in each cycle by the following questions:

- What content is effective for building conceptual mathematical knowledge?
- How does sequencing of content affect the content knowledge-building of the users?
- What elements of the app design engage users the most?
- How do users react to the design of visual elements of the software?
- What elements of the software increase or decrease engagement?
- What elements of the software increase or decrease anxiety in the user?
- Is the software stable for the user?

Data on users' experience will be the most salient information that can be gathered in these cycles. Their perceptions of the usefulness and usability of the intervention must guide development if the resulting software is to have any impact on the population. Therefore, the research methodologies used to investigate these questions will be:

- A review of the literature on mobile learning application development;
- Clinical interviews with members of the target audience to gauge the usability of the app;
- Clinical interviews with SMEs in undergraduate mathematics and mathematics education to assess the appropriateness of the nature and sequencing of the mathematical content;
- Survey of users to assess reaction in the affective domain;
- Case studies;

- Data analytics gathered by the software to determine the location and length of engagement and achievement levels of the users and the stability of the software.

This phase of development will be complete when the software is demonstrably stable, and answers that can be derived from individual users have been exhausted. At this point the project will be ready to situate in an undergraduate mathematics course.

Evaluation: Local Impact

The Evaluation: Local Impact phase will consist of a pilot study of the use of the instrument by students who are enrolled in an undergraduate mathematics for elementary teachers course. The guiding questions for this cycle of research will be:

- Do the students feel that the software supports the learning that is taking place in the course?
- Does the instructor perceive the software as a useable resource for the students?
- Do students continue to use the software throughout the course?
- Was there an effect on student achievement?
- Was there an effect on a the measure of mathematics knowledge for teaching?

The impact on student mathematics knowledge will determine if the software development cycles back to another enactment cycle or if it is is ready for wider dissemination. Research methodologies that may be used to investigate these questions are:

- MKT assessment;
- Measures of self-determination;
- Clinical interviews;
- Quasi-experimental studies;
- Focus groups;

- Data analytics.

What has been learned during the Local Impact stage will be leveraged to refine the software. With the completion of a successful Local Impact study the mobile application will be ready for broad dissemination.

Evaluation: Global Impact

The Evaluation: Global Impact phase will include assessments of user satisfaction and engagement. Guiding questions for this phase will include:

- What is the nature of user engagement with the software?
  - How long does each session last?
  - How long do users keep the app on their devices?
- Are users demonstrating self-determination in their use of the software?
  - What levels of achievement are they reaching in the software?
  - What additional topics do users request?

It is expected that at this point the scale of the project will necessitate quantitative research of user experience and perceptions. Appropriate research methodologies at this phase will include:

- In-app surveys
- Data analytics gathered through the application and user databases.

Challenges

No matter how well topics and contents are sequenced, it would be foolhardy to imagine that the complexity of learning mathematics can be reduced to the capabilities of a single mobile application. Still, leveraging the affordances that technology provides to sequence and present mass amounts of information and skill practice presents the possibility of developing a powerful tool to increase user experience, knowledge, and understanding. In spite of the fact that mobile devices have become ubiquitous in our world, research on mobile technology's impact on learners is in its infancy. There is a need for greater exploration of the impact of technologies that allow user autonomy in choosing where, when, and what learning will occur. Though these broad questions certainly cannot be answered in the development of a single mobile application, conducting this study will contribute to the body of knowledge in this domain.

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